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Network Topology-aware Traffic Scheduling

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Abstract

We propose a method for the optimal scheduling of collective data exchanges relying on the knowledge of the underlying network topology. The method ensures a maximal utilization of bottleneck communication links and offers an aggregate throughput close to the flow capacity of a liquid in a network of pipes. On a 32 node K-ring cluster we double the aggregate throughput by applying the presented scheduling technique. Thanks to the presented theory, for most topologies, the computational time required to find an optimal schedule takes less than 1/10 of a second.

Keywords: Optimal network utilization, traffic scheduling, all-to-all communications, collective operations, network topology, topology-aware scheduling.

1. Introduction

The interconnection topology is one of the key factors of a computing cluster. It determines the performance of the communications, which are often a limiting factor of parallel applications [1], [2], [3], [4]. Depending on the transfer block size, there are two opposite factors (among others) influencing the aggregate throughput. Due to the message overhead, communication cost increases with the decrease of the message size. However, smaller messages allow a more progressive utilization of network links. Intuitively, the data flow becomes liquid when the packet size tends to zero [5], [6]. The aggregate throughput of a collective data exchange depends on the underlying network topology and on the allocation of processing nodes to a parallel application. The total amount of data together with the longest transfer time across the most loaded links (*bottlenecks*) gives an estimation of the aggregate throughput. This estimation is defined here as the *liquid* throughput of the network. It corresponds to the flow capacity of a non-compressible fluid in a network of pipes [6]. Due to the packeted behaviour of data transfers, congestions may occur in the network and thus the aggregate throughput of a collective data exchange may be lower than the liquid throughput. The rate of congestions for a given data exchange may vary depending on how the sequence of transfers forming the data exchange is scheduled by the application.

The present contribution presents a scheduling technique for obtaining the liquid throughput. There are many other collective data exchange optimization techniques such as message splitting [7], [8], parallel forwarding [9], [10] and optimal mapping of an application-graph onto a processor graph [11], [12], [13]. Combining the above mentioned optimizations with the optimal scheduling technique described in the present article may be the subject of further research. There are numerous applications requiring highly efficient network resources: parallel acquisition of multiple video streams with successive contiguous all-to-all retransmission [14], [15], voice-over-data traffic switching [16], [17], high energy physics data acquisition and transmission from numerous detectors to a cluster of processing nodes for filtering and event assembling [18], [19].

Let us analyze an example of a collective data exchange on a simple topology (Fig. 1). Suppose that an *all-to-all* operation is taking place such that each of 5 transmitting processors sends an equal size packet to each of 5 receiving processors. Suppose the packet size is 1*MB* so that the data exchange operation transfers 25*MB* of data over the network.

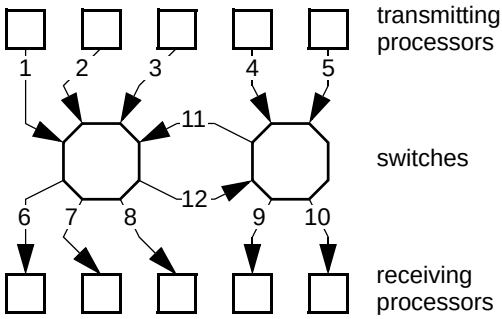


Fig. 1. Simple network topology.

During the collective data exchange, links 1 to 10 transfer $5MB$ of data each (Fig. 1). Links 11 and 12 are the bottlenecks and transfer $6MB$ each. Suppose that the throughput of a link is $100MB/s$. Since links 11 and 12 are the bottleneck, the longest transfer of the collective data exchange lasts $6MB/(100MB/s) = 0.06s$. Therefore the liquid throughput of the global operation is $25MB/0.06s = 416.67MB/s$. Let us now propose a schedule for successive data transfers and analyze its throughput.

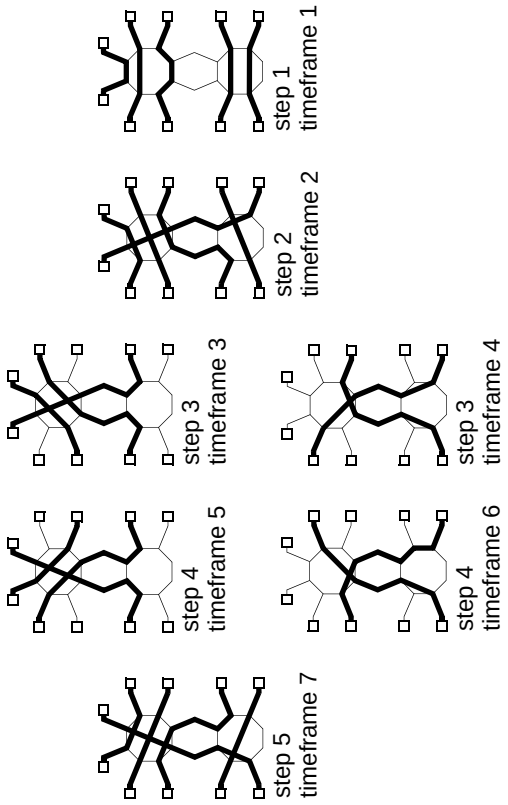


Fig. 2. Round-robin schedule of transfers.

Intuitively, a good schedule for an all-to-all exchange is a *round-robin* schedule where at each step each sender has a receiver shifted by one position. Let us now examine the round-robin schedule of an all-to-all data exchange on the network topology of figure 1. Figure 2 shows that logical steps 1, 2 and 5 can be processed in the timeframe of a single transfer. But logical steps 3 and 4 can not be processed in a single timeframe, since there are two transfers trying to simultaneously use the same links 11 and 12, causing a congestion. Two conflicting transfers need to be scheduled in two single timeframe substeps. Thus the round-robin schedule takes 7 timeframes instead of the expected 5 and accordingly, the throughput of the round-robin all-to-all exchange is:

$$25MB/\left(7 \times \left(\frac{1MB}{100MB/s}\right)\right) = 357.14MB/s.$$

It is therefore less than the liquid throughput (416.67*MB/s*). Can we propose an improved schedule for the all-to-all exchange such that the liquid throughput is reached?

By ensuring that at each step the bottlenecks are always used, we create an improved schedule, having the performance of the network's liquid throughput (Fig. 3). According to this improved schedule only 6 steps are needed for the implementation of the collective operation, i.e. the throughput is:

$$25MB/\left(6 \times \left(\frac{1MB}{100MB/s}\right)\right) = 416.67MB/s.$$

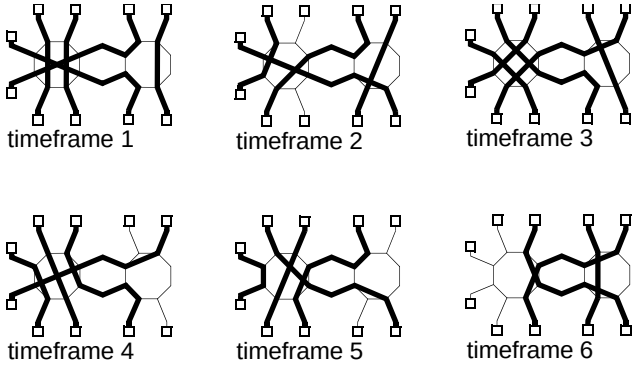


Fig. 3. An optimal schedule.

Section 2 shows how to describe the liquid throughput as a function of the number of contributing processing nodes and their underlying network topologies. An introduction to the formal theory of traffic scheduling is given in section 3. Section 4 presents measurements for the considered sub-topologies and draws the conclusions.

2. Throughput as a function of sub-topology

In order to evaluate the throughput of collective data exchanges we need to specify along an independent axis the number of processing nodes as well as significant variations of their underlying network topologies. To simplify the model let us limit the configuration to an identical number of receiving and transmitting processors forming successions of node pairs. The applications perform all-to-all data exchanges over the allocated nodes (each transmitting processor sends one packet to each receiving processor).

Let us demonstrate how to create variations of processing node allocations by considering the specific network of the Swiss-T1 cluster (called henceforth T1, see Fig. 4). The network of the T1 forms a K-ring [20] and has a static

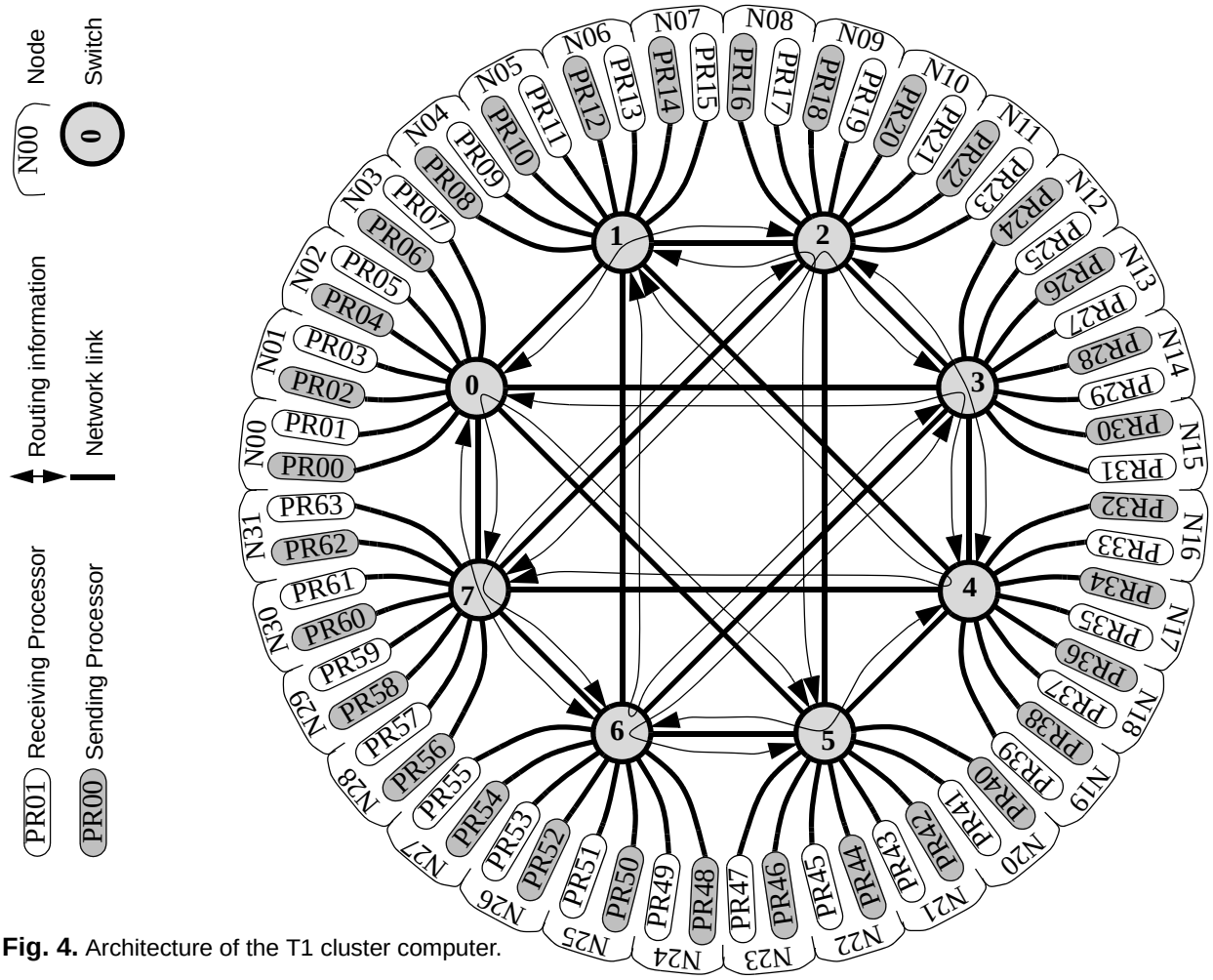


Fig. 4. Architecture of the T1 cluster computer.

routing scheme. The throughputs of all links are identical and are equal to 86*MB/s*. The cluster consists of 64 processors paired into 32 nodes [21], [22].

Since the T1 cluster incorporates 32 nodes, there exist 2^{32} = 4294967296 possible allocations of nodes to an application. Considering only the number of nodes in front of each switch, there are only 5^8 = 390625 different processing node allocations, since there are 8 switches having each n used nodes ($0 \le n \le 4$). Each allocation may be represented by a vector $\left[n_0\ n_1\ n_2\ n_3\ n_4\ n_5\ n_6\ n_7\right]$.

With a model incorporating the given network topology and routing tables, we can compute the liquid throughput of an all-to-all traffic for any allocation. The full set of 390625 allocation vectors is given as input to the model and the liquid throughput is computed for each input vector. For the T1's network, only 363 different values of liquid throughput are formed and thus the set of 390625 is partitioned into 363 different subsets. Each of the obtained 363 key sub-topologies is characterized by its liquid throughput and the number of allocated nodes (see Fig. 5). The figure demonstrates that depending on the sub-topology, the liquid throughput for a given number of nodes may considerably vary.

For the purpose of enumerating the 363 sub-topologies we sort these sub-topologies according to the number of nodes and within the same number of nodes according to the value of the liquid throughput. Figure 6 demonstrates the liquid throughput of the network together with the throughput of an imaginary full crossbar network. The horizontal axis represents the collection of the 363 sub-topologies together with the number of contributing processing nodes (in parentheses).

3. Liquid schedules

This section proposes a formal model of a collective data exchange. In this model a single point-to-point transfer is represented by the set of communication links forming the

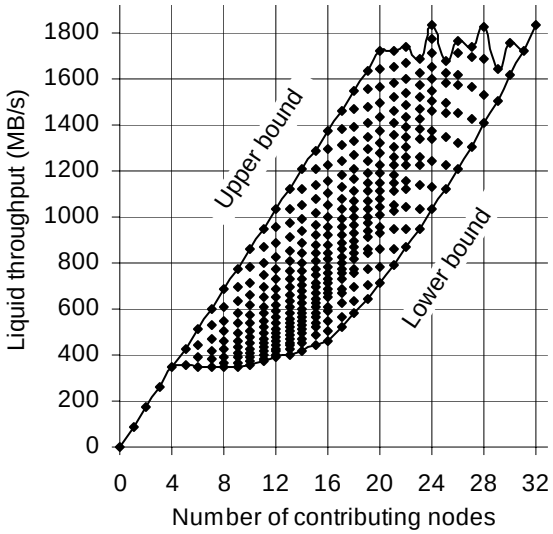


Fig 5. Liquid throughput in relation to the number of nodes with variations according to sub-topologies.

path between a transmitting and a receiving processor. The collective data exchange comprises a set of transfers having identical packet sizes. A sending processor may transfer a packet to a given receiving processor not more than once.

DEFINITIONS. A *transfer* is a set of links (i.e. the path from a sending processor to a receiving processor). A *traffic* is a set of transfers (see Fig. 7). A link l is *utilized* by a transfer x if $l \in x$. A link l is utilized by a traffic X if l is utilized by a transfer of X. Let a and b be transfers of a traffic X, the transfer b is in congestion with a, if b uses a link utilized by a. A sub-traffic of X (a subset of X) is *simultaneous* if it forms a collection of non-congesting transfers.

A simultaneous subset of a traffic is processed in the timeframe of a single transfer. The *load* of link l in the traffic X is the number of transfers in X using l. The *duration* $\Lambda(X)$ of a traffic X is the maximal value of the load among all links involved in the traffic. The links having maximal load values are called *bottlenecks*. The *liquid throughput* of a traffic X is the ratio $\#(X)/\Lambda(X)$ multiplied by a single link throughput, where $\#(X)$ is the number of transfers in the traffic X. For example, the traffic X shown in figure 7 has a number of transfers $\#(X) = 25$ and the duration of the traffic is $\Lambda(X) = 6$. Therefore the aggregate liquid throughput is the ratio $25/6$ of a single link throughput, i.e. $(25/6) \times 100 MB/s$, supposing a single link throughput of 100 *MB/s*.

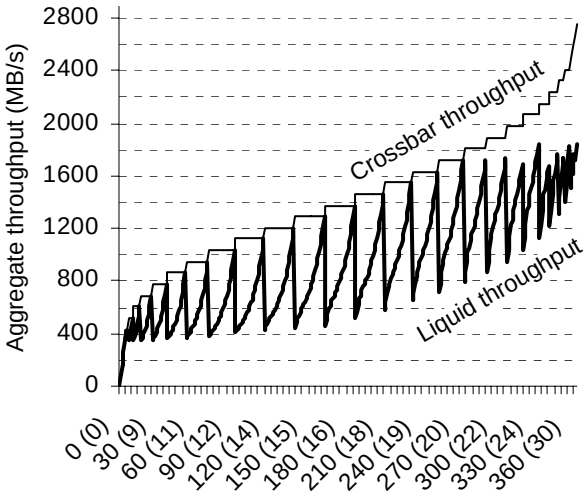
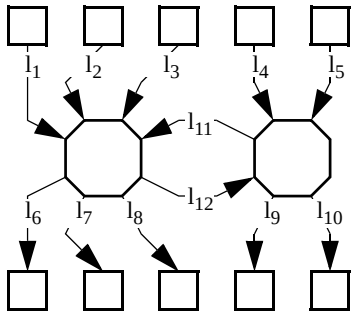


Fig. 6. Liquid and crossbar throughputs on T1.

Recall that a *partition* of X is a disjoint collection of non-empty subsets of X whose union is X [23]. A *schedule* α of a traffic X is a collection of simultaneous subsets of X partitioning the traffic X. A *timeframe* of a schedule α is an element of α. The *length* $\#(\alpha)$ of a schedule α is the number of timeframes in α. A schedule of a traffic is *optimal* if the traffic does not have any shorter schedule. If the length of a schedule is equal to the duration of the traffic then the schedule is *liquid*. A liquid schedule is optimal, but the inverse is not always true, meaning that a traffic may not have a liquid schedule (the demonstration is beyond the scope of this article). Figure 8 shows a liquid schedule of the collective traffic shown in figure 7.



$$\left\{ \begin{array}{c} \{l_1, l_6\}, \{l_1, l_7\}, \{l_1, l_8\}, \{l_1, l_{12}, l_9\}, \{l_1, l_{12}, l_{10}\}, \\ \{l_2, l_6\}, \{l_2, l_7\}, \{l_2, l_8\}, \{l_2, l_{12}, l_9\}, \{l_2, l_{12}, l_{10}\}, \\ \{l_3, l_6\}, \{l_3, l_7\}, \{l_3, l_8\}, \{l_3, l_{12}, l_9\}, \{l_3, l_{12}, l_{10}\}, \\ \{l_4, l_{11}, l_6\}, \{l_4, l_{11}, l_7\}, \{l_4, l_{11}, l_8\}, \{l_4, l_9\}, \{l_4, l_{10}\}, \\ \{l_5, l_{11}, l_6\}, \{l_5, l_{11}, l_7\}, \{l_5, l_{11}, l_8\}, \{l_5, l_9\}, \{l_5, l_{10}\} \end{array} \right\}$$

Fig. 7. All-to-all traffic.

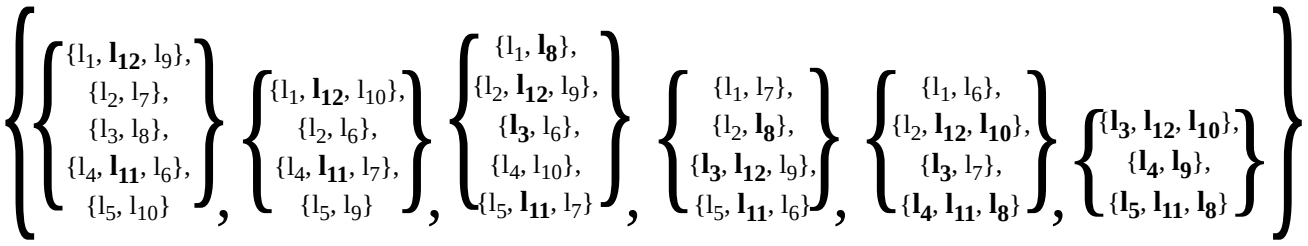


Fig. 8. A liquid schedule of the collective traffic shown in figure 7 (see also figure 3).

The duration of a traffic X is the load of its bottlenecks. Consider l as one of the bottlenecks of X. The load of l is the number of transfers in X using l. Now let α be a schedule on X. By definition α is a collection of simultaneous subsets of X, partitioning X. Since α partitions X, a transfer of X (using l) shall be found in one and only one of the timeframes of α. Since a timeframe of α is simultaneous it may contain only one or no transfer using l. Therefore if the length of α is equal to the number of transfers in X using the bottleneck l, then each timeframe of α shall contain a transfer using l. Inversely, if each timeframe of α has a transfer using l, then the length of α shall be equal to the number of transfers using l. Hence if a schedule is liquid then each of its timeframes uses all bottlenecks, and if all timeframes of a schedule use all bottlenecks then the schedule is liquid.

In other words, we derived an equivalent condition for the liquidity of a schedule. The necessary and sufficient condition for the liquidity of a schedule is that all bottlenecks be used by each timeframe of the schedule. Let us define a simultaneous subset of X as a *team* of X if it uses all bottlenecks of X. Consequently, an equivalent condition for the liquidity of a schedule α on X is that each timeframe of α be a team of X.

Our goal is to design an algorithm that may partition a traffic so as to form a liquid schedule (whenever possible).

DISCUSSION. Suppose A is a timeframe of a liquid schedule α on a traffic X. Therefore A is a team of α. Remove the team A from X so as to form a new traffic $X-A$. The duration of the new traffic $X-A$ is the load of the bottlenecks in $X-A$. The bottlenecks of X are bottlenecks of $X-A$. The load of a bottleneck of X decreases by one in the new traffic $X-A$. However the new traffic $X-A$ may have additional bottlenecks. The schedule α without the element A is a schedule for $X-A$ with the previous length decreased by one. The new schedule $\alpha-\{A\}$ has as many timeframes as the duration of the new traffic $X-A$. Therefore $\alpha-\{A\}$ is a liquid schedule on $X-A$.

In other words, if the traffic has a liquid schedule, then a schedule reduced by one team is a liquid schedule on the reduced traffic.

This is the key point in searching for a liquid schedule. Consider traffic X as a problem whose solution is a liquid schedule α. Assume a technique capable of generating the set of all teams of X. If X has a solution α then a timeframe A of the schedule α is a member of the set of all teams of X and $\alpha-\{A\}$ is a schedule on $X-A$. Therefore the problem X can be reduced into smaller problems. Examine each possible team A of X and search inductively (e.g. recursively) a solution for $X-A$. If a solution exists for X, then the method will find it. If the method does not find a solution for X, and since we explored the full solution space, we conclude that X does not have a liquid schedule.

We limit at each iteration our choice to the collection of only those teams of the original traffic which are also teams of the current reduced sub-traffic. By doing so, we considerably reduce the search space without affecting the solution space.

DEFINITIONS. A simultaneous subset A of a traffic X is *full* with respect to X if each transfer of $X-A$ is in congestion with a transfer of A. A team of X is called *full* team if it is a full simultaneous subset of X.

We intend to limit the search space when building a liquid schedule. Let us modify a liquid schedule so as to convert one of its teams into a full team. Let X (a traffic) have a solution α (a liquid schedule). Let A be a timeframe of α. If A is not a full team of X, then, by moving the necessary transfers from other timeframes of α, we can convert timeframe A to a full team. Evidently, the properties of liquidity (partitioning, simultaneousness and length) of α will not be affected. Therefore if X has a solution then it has also a solution when one of its timeframes is full, hence the choice of the teams in the construction may be narrowed from the set of all teams to the set of full teams only.

By a choice of a full team A of a traffic X we are faced with the new smaller problem of searching a liquid schedule for

a traffic $X - A$. The traffic $X - A$ may not have a solution, or it may not have even a team. In these cases we have to backtrack to evaluate other choices. Evaluation of all choices ultimately leads to a solution if it exists.

Figure 8 shows a liquid schedule built as explained above. Let us denote the timeframes in figure 8 as follows: $\{A_1, A_2, A_3, A_4, A_5, A_6\}$, according to the order given in figure 8. Traffic X is the union of the timeframes $X = \bigcup_{i=1}^{6} A_i$. The schedule is constructed such that at any step i, the timeframe A_i is a full team of the sub-traffic $X - \bigcup_{k=1}^{i-1} A_k$. The timeframe A_i being a team of the sub-traffic $X - \bigcup_{k=1}^{i-1} A_k$ incorporates therefore all bottlenecks of this sub-traffic (shown in bold).

In order to be able to explore the full solution space for obtaining a liquid schedule, we need to successively build all full teams. We designed a procedure capable of generating (without repetitions) all successive full teams for an arbitrary traffic. It first builds *skeletons*, an intermediate collection of teams from a sub-traffic including only those transfers which comprise bottlenecks. Then it extends each skeleton by applying variations of all non-congesting transfers in order to build up all full teams.

5. Results and conclusion

For an unknown network topology only two intuitively reasonable schedules make sense, the round-robin and the random schedule. Measurements of the round-robin schedule and of the random schedule show a similar throughput. The round-robin schedule on a T1 cluster is shown in figure 9. The amount of data transferred from one processor to another processor is equal to 2*MB* and the transfer block size is 520*KB*. The figure presents the result of 4344 measurements of all-to-all data exchanges. For each topology, 20 measurements were performed. The median of the collected results is represented as a small black square. The thick curve represents the liquid throughput. The thin curve below the graph of the liquid throughput represents the theoretically predicted throughput of the round-robin schedule, computed for the model of the T1 cluster.

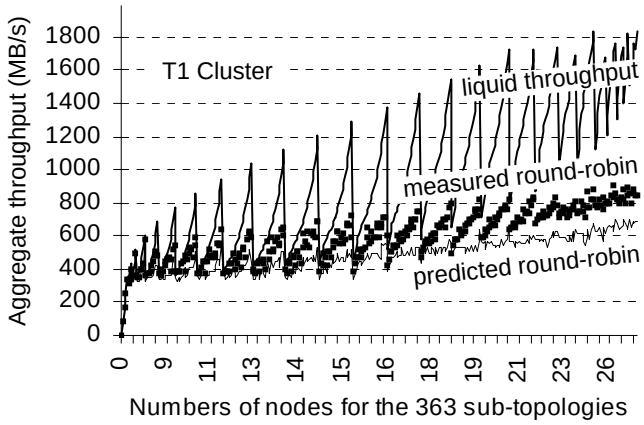


Fig. 9. Throughput of the round-robin schedule.

The measured throughput is higher than the predicted throughput. This increase in throughput in the real network is explained by a “semifluid” behaviour of transfers: time shifts in packet transfers tend to reduce congestions. Nevertheless the measured round-robin throughput for a large number of sub-topologies is only 50% of the liquid throughput.

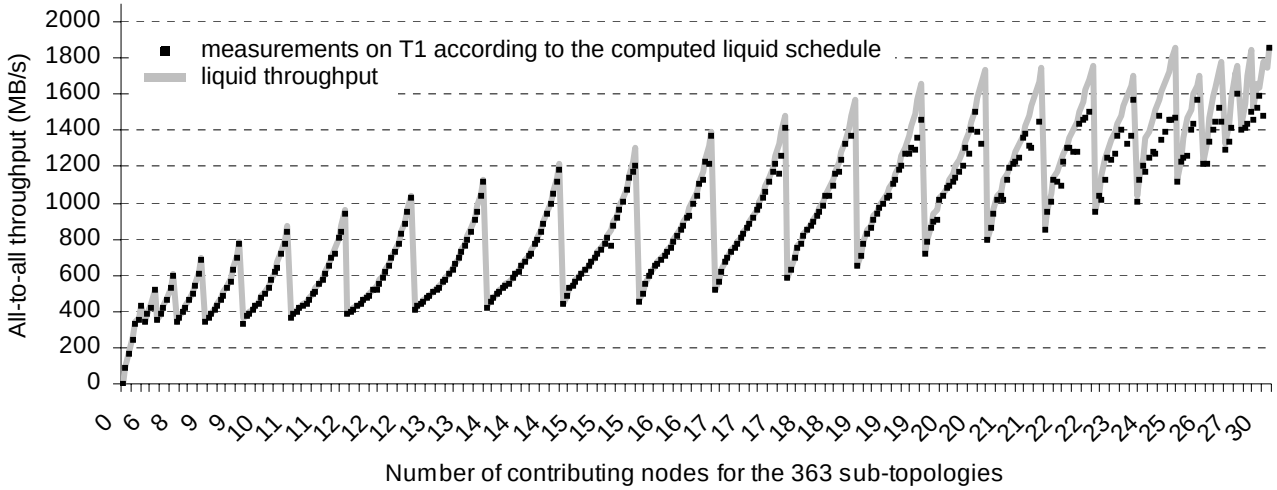


Fig. 10. Predicted liquid throughput and measured throughput according to the computed liquid schedule.

Thanks to the presented theory, we strongly reduce the search space of liquid schedules. The computation time of a liquid schedule takes for more than 97% of the considered sub-topologies of the T1 cluster less than 1/10 of a second on a single 500MHz Alpha processor. Figure 10 shows the measured aggregate throughput of an all-to-all collective traffic executed on T1, optimized by applying our liquid schedule based traffic partitioning technique. Each black dot represents the median of 7 measurements. The horizontal axis represents the 363 sub-topologies as well as the number of contributing nodes. Processor to processor transfers have a size of 5*MB*, transferred as a single message of 5*MB*. The measured all-to-all aggregate throughputs (black dots) are close to the theoretically computed liquid throughput (gray line). For many sub-topologies, the proposed liquid scheduling technique allows to increase the aggregate throughput by at least a factor of two compared with a simple round-robin or random schedule.

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