The Theory of the Moiré Phenomenon

by

Isaac Amidror

Peripheral Systems Laboratory,
Ecole Polytechnique Fédérale de Lausanne,
Lausanne, Switzerland

KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

2000
If the impulse whose frequency vector is \( f \) falls inside the visibility circle and represents a visible moiré in the superposition of the \( m \) original images, the above formulas (2.8) express the frequency, the period and the angle of this moiré. Note that, as shown in Sec. C.1 of Appendix C, in the special case of \( m = 2 \), where a moiré effect occurs due to the vectorial sum of the frequency vectors \( f \) and \(-f\) (see Fig. 2.2), these formulas are reduced to the familiar geometrically obtained formulas of the period and angle of the moiré effect between two gratings [Nishijima64]:

\[
T_u = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2 - 2T_1 T_2 \cos \alpha}} \quad \varphi_u = \arctan \left( \frac{T_2 \sin \theta_1 - T_1 \sin \theta_2}{T_1 \cos \theta_1 - T_2 \cos \theta_2} \right)
\] (2.9)

where \( T_1 \) and \( T_2 \) are the periods of the two original gratings and \( \alpha \) is the angle difference between them, \( \theta_1 - \theta_2 \). (Note, however, that these formulas are only valid when \( T_1 = T_2 \); the reason for this restriction will become clear at the end of Sec. 2.6.) In the particular case where \( T_1 = T_2 \) this is further simplified into the well-known formulas [Nishijima64]:

\[
T_u = \frac{T}{2 \sin (\alpha/2)} \quad \varphi_u = 90^\circ + \frac{1}{2}(\theta_1 + \theta_2)
\] (2.10)

Note that in this case the direction of the moiré is perpendicular to the bisector of the angle formed between the grating directions. In another interesting particular case where \( T_1 \neq T_2 \) but \( \theta_1 = \theta_2 \) (namely: the superposition of two parallel gratings) Eqs. (2.9) are reduced into the equally well-known formulas:

\[
T_u = \frac{T_1 T_2}{|T_1 - T_2|} \quad \text{(i.e.,} \quad f_u = |f_1 - f_2|) \quad \varphi_u = \theta
\] (2.11)

Eqs. (2.7), (2.8) and their derived formulas (2.9)–(2.11) give the geometric properties of an impulse in the spectrum of the superposition (and of the periodic component or moiré that it represents in the image domain), namely, the period and the direction. The amplitude of any individual impulse, which represents the strength of the corresponding periodic component in the image, is a product of the amplitudes of the \( m \) impulses from which it has been obtained in the convolution, one from each of the \( m \) spectra:

\[
a = a_1 \cdot \ldots \cdot a_m
\]  (2.12)

Note, however, that if two or more impulses in the convolution happen to fall on top of each other exactly in the same location, their individual amplitudes are summed.

As we can see from Eqs. (2.7) and (2.12), the convolution of impulsive spectra can be considered as an operation in which frequency vectors of the original spectra are added vectorially, whereas the corresponding impulse amplitudes are multiplied. These rules follow from the properties of convolution, and they can be readily verified by the “move and multiply” method. Note that if all the convolved spectra are real-valued and symmetric about the origin (see Sec. 2.2), the resulting spectrum is also real-valued and symmetric.

---

11 Note that the moiré angle formulas found in literature may vary according to the angle conventions being used.
References


