Actually, the moiré effect is not limited in any way to separate rotations and elongations nor to infinitesimal deformations. Neither is it necessary that the arrays be originally identical in either spacing or orientation.

In this paper only sets of straight, parallel, nondiffracting lines will be considered. However, moiré fringes that can be analyzed could be obtained from circular, radial, or other nonparallel arrays, and diffraction gratings could be used under suitable conditions.

Fig. 1(c) shows the effect of combined rotation and differences in pitch of straight, parallel sets of lines representing a homogeneous field for which equations will be derived. The relationships so obtained can also be applied to sufficiently small elements of a nonhomogeneous field such as would be presented in a general two-dimensional strain problem.

The moiré method can certainly be used for the analysis of large strains and rotations, but such application is beyond the scope of this paper. However, the equations will be derived in the most general terms before being related to such specific quantities as strain. Approximations for the case of infinitesimal deformations will also be developed.

**BASIC PROPERTIES AND DEFINITIONS**

In the subsequent derivations, a fixed array of straight, parallel lines called the “master grid” is used as a reference both for analysis of the properties of the moiré and for establishment of coordinate directions. The center-to-center distance between the master grid lines is defined as the master pitch, \( p \), and the directions perpendicular and parallel to these lines are designated as \( r \) and \( s \), respectively. The model also bears a similar array of lines which in the undistorted state are not necessarily of the same pitch as the master grid. The pitch of this grid at any particular point in any state of distortion is called the “model pitch” and is designated by \( p' \).

**Inclination of the Fringes.**

\[
\overline{AB} = \frac{p}{\cos \left( \phi - \frac{\pi}{2} \right)} = \frac{p}{\sin \phi} \quad \quad \quad \quad (1)
\]

and

\[
\overline{AB} = \frac{p'}{\cos \left( \phi - \frac{\pi}{2} - \theta \right)} = \frac{p'}{\sin (\phi - \theta)} \quad \quad \quad \quad (2)
\]

therefore

\[
p' = p \frac{\sin (\phi - \theta)}{\sin \phi} \quad \quad \quad \quad (3)
\]

Because

\[
p' (\sin \phi \cos \theta - \sin \theta \cos \phi) = p' \sin \phi \quad \quad \quad \quad (4)
\]

then

\[
\tan \phi = \frac{p \sin \theta}{p \cos \theta - p'} \quad \quad \quad \quad (5)
\]

**Distance between Fringes.**—Also from Fig. 2 and the foregoing:

\[
a = \frac{p}{\sin \theta} \quad \quad \quad \quad \quad \quad \quad \quad \quad (6)
\]

and

\[
\delta = a \cos \left( \phi - \frac{\pi}{2} - \theta \right) = \frac{p \sin (\phi - \theta)}{\sin \theta} \frac{p'}{\sin \phi} \quad \quad \quad \quad (7)
\]
\[
\sin \theta = \frac{\phi \cos \theta + d}{\rho_0}
\]

Formulation:

1. Substitute Eq. 2 into Eq. 11 and extract sin \theta from trigonometric terms.
2. \( \phi = \frac{\rho_0}{\sin \theta} \)
3. \( \phi \sin \theta = \rho_0 \)
4. \( \phi \sin \theta = \rho_0 \)

Two cases (a) \( \rho_0 = 0 \), and (b) \( \rho_0 \neq 0 \), respectively:
1. Case (a): \( \rho_0 = 0 \)
2. Case (b): \( \rho_0 \neq 0 \)

For the method of describing the model of the fringe, the model will be described in a separate section.

For the definition of the fringe, the model of the fringe is described in a separate section.

Case (a) \( \rho_0 = 0 \):
\[
\frac{d}{d} = \frac{d}{d} = \frac{d}{d} = \frac{d}{d}
\]

Case (b) \( \rho_0 \neq 0 \):
\[
\frac{d}{d} = \frac{d}{d} = \frac{d}{d} = \frac{d}{d}
\]

II Fig. 1 is converted to the form

The master fringe line is located at \( d \neq 0 \) and \( d \neq 0 \), which determines the direction of the fringe. The fringe direction is \( d \neq 0 \).

The fringe direction is \( d \neq 0 \).

Values of \( d \) are not possible values of \( d \) and \( d \) to show the total fringe of the model which is a method of describing the fringe.

Case (a) \( \rho_0 = 0 \):
\[
\frac{d}{d} = \frac{d}{d} = \frac{d}{d} = \frac{d}{d}
\]

Case (b) \( \rho_0 \neq 0 \):
\[
\frac{d}{d} = \frac{d}{d} = \frac{d}{d} = \frac{d}{d}
\]
\[
\left(309\right) \quad \left(1 + \frac{d}{d} \cos \left(\frac{d}{d} + \gamma \right)\right) \frac{d}{d} = \frac{d}{d} \sin \theta
\]

and

\[
\left(310\right) \quad \left(1 - \frac{d}{d} \right) \frac{d}{d} = \frac{d}{d}
\]

Then

\[
\left(27\right) \quad \phi \cos \left(\frac{d}{d} \sin \theta\right) + 1 = \frac{d}{d}
\]

and

\[
\left(28\right) \quad \phi \cos \left(\frac{d}{d} \sin \theta\right) = \frac{d}{d}
\]
EQUATION ANALYSIS

USE OF THE BASIC RELATIONSHIPS IN TWO-DIMENSIONAL

...
where $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ are the second derivatives of $f$ with respect to $x$ and $y$, respectively.

The values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are calculated using finite differences at the points of interest to determine the rate of change in $f$. The second derivatives are calculated using the central difference formula:

$$\frac{\partial^2 f}{\partial x^2} \approx \frac{f(x+h, y) - 2f(x, y) + f(x-h, y)}{h^2}$$

$$\frac{\partial^2 f}{\partial y^2} \approx \frac{f(x, y+h) - 2f(x, y) + f(x, y-h)}{h^2}$$

where $h$ is a small value. 

The process of differentiation allows the determination of the slope of the function at any point of interest. This is particularly useful in understanding the behavior of the function near critical points or boundaries.

Since $\phi$ is a function of $x$ and $y$, the derivatives $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ provide information about the rate of change of $\phi$ with respect to $x$ and $y$, respectively. The second derivatives help to identify points of inflection or curvature within the function.

The figures illustrate the distribution of these derivatives, with the radial fringes indicating the variation of $f$ across the observed area. The fringes are more prominently observed where the derivatives are large, indicating significant changes in the function's behavior.

The process can be extended to higher dimensions, allowing for the analysis of functions in multiple variables. This technique is fundamental in various fields such as physics, engineering, and economics, where understanding the behavior of complex functions is crucial.
APPLICATION OF THE METHOD TO AN ACTUAL PROBLEM

(\(\frac{x'}{d} - \frac{x}{d}\)) = \lambda

\(x' = x - \lambda d\)

\(x = x' + \lambda d\)

The method discussed here can be applied to the familiar case of a circular disk.

The disk was loaded between the plates, first with grid lines perpendicular to the loading direction (Fig. 11). Then the disk was loaded along the grid lines in the direction of the grid lines. In the loading direction, (Fig. 10), the lines of strain are assumed to be the dark lines between the plates, first with grid lines perpendicular to the loading direction.
Fig. 13.—Principal Stresses on the horizontal axis of a disk (Hysol 8705) under diametral compression, obtained using the Moiré method.

NOTE: AREA UNDER $\sigma_2$ CURVE IS 1.01 $\sigma_0 = \frac{P}{D\theta}$

From a series of measurements across the horizontal axis of Fig. 10, the value of $\sigma_1$ was determined at a number of points. These values, combined with those determined by the Moiré fringes method and given in Fig. 12 and with the values of $\sigma_2$ obtained graphically in Fig. 13, are the results of the experiment. The difference between the theoretical and experimental values can be attributed mainly to the flattening of the disk by the ink plates used to apply the load.

ACKNOWLEDGMENTS

The writers wish to acknowledge the financial support received from the National Science Foundation and from the Armour Research Foundation of the Illinois Institute of Technology.